CIEG436	S2023			Total enrollment	23		
HW5 #3 (liner design)				Total env majors	16	Total civil majors	3
Outcome 2 - design				Env majors assessed	16	Civil majors assessed	3
n =	19	(16 env, 3 civ)					
Individual	Scores	Notes					
1	3	env					
2	4	env					
3	4	env					
4	4	env					
5	2	env					
6	4	env					
7	1	env					
8	4	env					
9	3	env					
10	3	env					
11	3	env					
12	4	env					
13	4	env					
14	4	env					
15	3	env					
16	3	env					
17	2	civil					
18	4	civil					
19	2	civil					
	ENV		CIV				
	3.3125	**average**	2.666667				
	0.873212	**SD**	1.154701				

CIEG436 S2023 HW5 #3 for ABET Outcome 2

3. Contaminant movement through liner (10 pts)

A geomembrane manufacturer has a new, thin (30 mil) geomembrane that has a never-before-seen hydraulic conductivity of 10^{-14} cm/sec. Your usual 60 mil GM has a hydraulic conductivity of 10^{-13} cm/s. You're interested in using the thinner GM in your 50-acre landfill because you believe it will be easier to work with; however, you suspect you may need additional clay below the GM. The clay contains a significant amount of bentonite and has a hydraulic conductivity of 10^{-8} cm/s.

You decide to compare 2 options for liner system design:

	Option A	**Option B**
Geomembrane	30 mil	60 mil
Clay	3.5 ft	3ft

Your goal is to design to be protective for a 12-inch layer of leachate, containing 3 mg/L of trichloroethylene (TCE), on top of the liner. The effective diffusion coefficient of TCE through the geomembrane material is $3{\times}10^{-9}$ cm^2/s and the effective diffusion coefficient of TCE through the clay is $2{\times}10^{-6}$ cm^2/s. Assume that the clay is saturated, the water pressure below the liner is equal to atmospheric pressure, the leachate liquid level above the geomembrane remains constant, and that the concentration of TCE at the bottom of the clay is zero. Follow the analysis in the lecture notes and in Box 10.8.3 in Chapter 10.8 of Christensen (2011). *1 mil = 0.001 inches.*

Your 50-acre landfill cannot contribute more than 2 kg/year of TCE to the groundwater from advective and diffusive flux together. Which option would you select to meet this requirement?

Option A	30	mil GM	0.0762 cm
	3.5	ft clay	106.68 cm
	1.00E-14	GM K, cm/s	
	1.00E-08	clay K, cm/s	
	3.00E-09	GM D, cm2/s	
	2.00E-06	clay D, cm2/s	

137.2362 delta h
106.7562 delta L
106.7562 delta z

Keq 1.4E-11 cm/s
Deq 1.36E-06 cm2/s

Advection
5.39545E-11 mg*cm/L*sec
0.067982624 g/acre/year
3.399131216 g/year over 50 acre landfill

Diffusion
3.81001E-08 mg*cm/L*sec
48.00609601 g/acre/year
2400.304801 g/year over 50 acre landfill

Total
2403.703932 g/year
2.403703932 kg/year

Option B	60	mil GM	0.1524 cm
	3	ft clay	91.44 cm
	1.00E-13	GM K, cm/s	
	1.00E-08	clay K, cm/s	
	3.00E-09	GM D, cm2/s	
	2.00E-06	clay D, cm2/s	

122.0724 delta h
91.5924 delta L
91.5924 delta z

Keq 5.97416E-11 cm/s
Deq 9.48947E-07 cm2/s

Advection
2.38867E-10 mg*cm/L*sec
0.300972167 g/acre/year
15.04860835 g/year over 50 acre landfill

Diffusion
3.10816E-08 mg*cm/L*sec
39.1628678 g/acre/year
1958.14339 g/year over 50 acre landfill

Total
1973.191998 g/year
1.973191998 kg/year

TCE conc	3	mg/L	
liquid	12	" head	30.48 cm head
LF size	50	acres	

40000000	cm2/acre
31500000	sec/year
1000	g/kg; cm3/L

1

#3

12 inch leachate = 30.48 cm

60 mil GM = 0.06" = 0.15 cm = t_{GM}

3 ft clay = 91.44 cm = t_{clay}

C = 3 mg/L TCE

$$K_{eq} = \frac{t_{GM} + t_{clay}}{\frac{t_{GM}}{K_{GM}} + \frac{t_{clay}}{K_{clay}}} = \frac{0.15 + 91.44}{\frac{0.15}{10^{-13}} + \frac{91.44}{10^{-8}}} = 6.07 \times 10^{-11} \text{ cm/s}$$

$$D_{eq} = \frac{t_{GM} + t_{clay}}{\frac{t_{GM}}{D_{GM}} + \frac{t_{clay}}{D_{clay}}} = \frac{0.15 + 91.44}{\frac{0.15}{3 \times 10^{-9}} + \frac{91.44}{2 \times 10^{-6}}} = 9.57 \times 10^{-7} \text{ cm}^2\text{/s}$$

Δh = top of leachate → bottom of clay
= 122 cm (approx)

$\Delta L = t_{GM} + t_{clay} = 91.59$ cm $= \Delta z$

Advective flux $= K_{eq}\left(\frac{\Delta h}{\Delta L}\right) C = 6.07 \times 10^{-11}\left(\frac{122}{91.59}\right)(3)$

$= 2.43 \times 10^{-10}$ mg cm/LS

$\times 4 \times \frac{10^7 \text{ cm}^2}{ac} \times \frac{3.15 \times 10^7 \text{ s}}{yr}$ / 1000/1000

= 0.30618 g/ac/yr × 60 ac LF

= 15.309 g/yr

Diffusive flux $= D_{eq}\left(\frac{\Delta C}{\Delta z}\right) = 9.57 \times 10^{-7}\left(\frac{3}{91.59}\right)$

$= 3.13 \times 10^{-8}$ mg cm/Ls

$= 4.98 \times 10^{-4}$ g/ac/yr × 50 ac

= 0.25 g/yr

15.309 + 0.25 = 15.559 g/yr = 0.02 kg/yr
which is well under requirement of 72 kg/yr

(2)

(3)

new: 30mil GM $K = 10^{-14}$ cm/sec
usual: 60mil GM $K = 10^{-13}$ cm/s

50 acre field clay $K = 10^{-8}$ cm/s

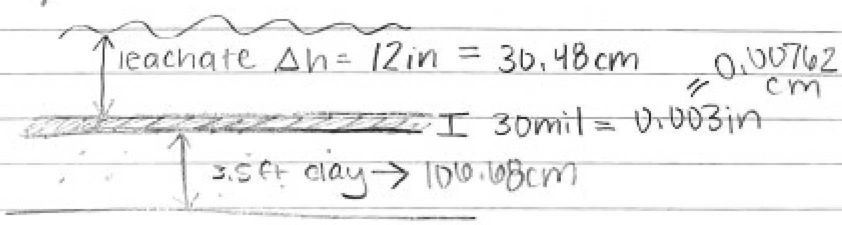
	OPTION A	OPTION B
GM	30mil	60mil
Clay	3.5ft	3ft

1mil = 0.001in

goal: 12in layer leachate
↳ 3mg/L TCE → diffusion coeff GM → $3\times10^{-9}\ \frac{cm^2}{s}$
$[TCE]_{bottom} = \emptyset$ P = Patm ↘ clay = $2\times10^{-6}\ \frac{cm^2}{s}$

- cannot exceed 2kg/year of TCE to the groundwater from advective + diffusive flux together
- Which option?

[Option A]



$$K_{eq} = \frac{t_{GM} + t_{clay}}{\frac{t_{GM}}{K_{GM}} + \frac{t_{clay}}{K_{clay}}} = \frac{0.00762cm + 106.68cm}{\frac{0.00762cm}{10^{-14}cm/s} + \frac{106.68cm}{10^{-8}cm/s}} = 1.38\times10^{-10}\ cm/s \ \#$$

$$D_{eq} = \frac{t_{GM} + t_{clay}}{\frac{t_{GM}}{D_{GM}} + \frac{t_{clay}}{D_{clay}}} = \frac{0.00762cm + 106.68cm}{\frac{0.00762cm}{3\times10^{-9}\frac{cm^2}{s}} + \frac{106.68cm}{2\times10^{-6}\frac{cm^2}{s}}}$$

$$= 1.91\times10^{-6}\ cm^2/s \ \#$$

$\Delta L = t_{GM} + t_{clay} = 106.68762\ cm$

$$\left(1.38\times10^{-10}\frac{cm}{s}\right)\left(\frac{137.16\,cm}{106.68762\,cm}\right)\left(3\frac{mg}{L}\right)$$

$$= 5.32\times10^{-10}\ \frac{cm\cdot mg}{L\cdot s}$$

$$\left(5.32\times10^{-10}\right)\left(\frac{4\times10^{7}cm^{2}}{acre}\right)\left(\frac{3.15\times10^{7}s}{yr}\right)\left(\frac{1g}{1000mg}\right)\left(\frac{1L}{1000cm^{3}}\right)$$

$= 0.745\ g/acre/yr$ #

diffusive flux $= D_{eq}\left(\frac{\Delta C}{\Delta z}\right)$

$\Delta z = t_{GM} + t_{clay} = 106.68762\,cm$

$$= \left(1.91\times10^{-6}cm^{2}/s\right)\left(\frac{3.3-0\frac{mg}{L}}{106.68762\,cm}\right) = 5.91\times10^{-8}\ \frac{mg}{s\cdot L}$$

③ CONT **OPTION A**

advective flux $= qC = K_{eq}\left(\frac{\Delta h}{\Delta L}\right)C$

$\Delta h =$ top of leachate + bottom of clay $= 30.48\text{cm} + 106.68\text{cm}$
$= 137.16\text{ cm}$

$\Delta L = t_{GM} + t_{clay} = 106.68762\text{ cm}$

$$\left(1.38\times10^{-10}\frac{\text{cm}}{\text{s}}\right)\left(\frac{137.16\text{cm}}{106.68762\text{cm}}\right)\left(3\frac{\text{mg}}{\text{L}}\right)$$
$$= 5.32\times10^{-10}\ \frac{\text{cm}\cdot\text{mg}}{\text{L}\cdot\text{s}}$$

$$(5.32\times10^{-10})\left(\frac{4\times10^{7}\text{cm}^2}{\text{acre}}\right)\left(\frac{3.15\times10^{7}\text{s}}{\text{yr}}\right)\left(\frac{1\text{g}}{1000\text{mg}}\right)\left(\frac{1\text{L}}{1000\text{cm}^3}\right)$$
$= 0.745$ g/acre/yr #

diffusive flux $= D_{eq}\left(\frac{\Delta C}{\Delta z}\right)$

$\Delta z = t_{GM} + t_{clay} = 106.68762\text{cm}$

$$= (1.91\times10^{-6}\text{cm}^2/\text{s})\left(\frac{3.3-0\ \frac{\text{mg}}{\text{L}}}{106.68762\text{cm}}\right) = 5.91\times10^{-8}\ \frac{\text{mg}}{\text{s}\cdot\text{L}}$$

$$5.91\times10^{-8}\frac{\text{mg}}{\text{s}\cdot\text{L}}\left(\frac{3.15\times10^{7}\text{s}}{\text{yr}}\right)\left(\frac{1\text{g}}{1000\text{mg}}\right)\left(\frac{1\text{L}}{1000\text{cm}^3}\right)\left(\frac{4\times10^{7}\text{cm}^2}{\text{acre}}\right)$$

$= 82.74$ g/acre/year #

Total flux $= 83.485$ g/acre/year

$= 0.083$ kg/acre/yr (50 acres)

$= 4.17$ kg/year

$$K_{eq} = \frac{t_{GM} + t_{clay}}{\frac{t_{GM}}{K_{GM}} + \frac{t_{clay}}{K_{clay}}} = \frac{0.1524\,cm + 91.44\,cm}{\frac{0.1524\,cm}{10^{-13}\,cm/s} + \frac{91.44\,cm}{10^{-8}\,cm/s}}$$

$$= \underline{5.97 \times 10^{-11}\ cm/s}\ \#$$

$$D_{eq} = \frac{t_{GM} + t_{clay}}{\frac{t_{GM}}{D_{GM}} + \frac{t_{clay}}{D_{clay}}} = \frac{0.1524 + 91.44}{\frac{0.1524}{3\times10^{-9}\,\frac{cm^2}{s}} + \frac{91.44}{2\times10^{-10}\,\frac{cm^2}{s}}}$$

$$= \underline{9.49 \times 10^{-7}\ cm^2/s}\ \#$$

<u>Advective flux</u> = $K_{eq}(\Delta h/\Delta L)C$

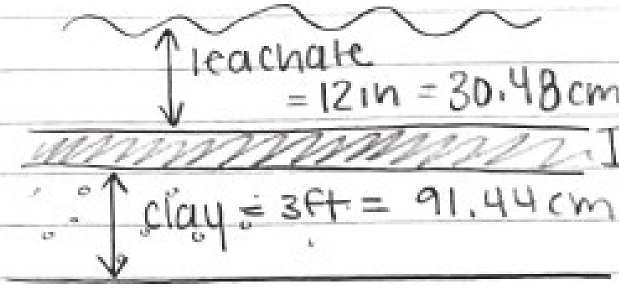
Δh = top leachate + bottom of clay = 30.48 cm + 91.44 cm
= 121.92 cm

$\Delta L = t_{GM} + t_{clay}$ = 91.5924 cm

$$\left(5.97\times10^{-11}\,\frac{cm}{s}\right)\left(\frac{121.92\,cm}{91.5924\,cm}\right)\left(3\,\frac{mg}{L}\right) = 2.38\times10^{-10}\ \frac{cm\cdot mg}{L\cdot s}$$

= 0.3003 g/acre/year

③ CONT **OPTION B**



$K = 10^{-13}$ cm/s

60 mil = $0.06\text{in}\left(\frac{2.54\text{cm}}{\text{in}}\right)$

= 0.1524 cm

$$K_{eq} = \frac{t_{GM} + t_{clay}}{\frac{t_{GM}}{K_{GM}} + \frac{t_{clay}}{K_{clay}}} = \frac{0.1524\text{cm} + 91.44\text{cm}}{\frac{0.1524\text{cm}}{10^{-13}\text{cm/s}} + \frac{91.44\text{cm}}{10^{-8}\text{cm/s}}}$$

$$= \underline{5.97 \times 10^{-11}\ \text{cm/s}}\ \#$$

$$D_{eq} = \frac{t_{GM} + t_{clay}}{\frac{t_{GM}}{D_{GM}} + \frac{t_{clay}}{D_{clay}}} = \frac{0.1524 + 91.44}{\frac{0.1524}{3\times10^{-9}\frac{\text{cm}^2}{\text{s}}} + \frac{91.44}{2\times10^{-10}\frac{\text{cm}^2}{\text{s}}}}$$

$$= \underline{9.49 \times 10^{-7}\ \text{cm}^2/\text{s}}\ \#$$

Advective flux = $K_{eq}(\Delta h / \Delta L)C$

Δh = top leachate + bottom of clay = 30.48 cm + 91.44 cm = 121.92 cm

$\Delta L = t_{GM} + t_{clay}$ = 91.5924 cm

$$\left(5.97\times10^{-11}\frac{\text{cm}}{\text{s}}\right)\left(\frac{121.92\text{cm}}{91.5924\text{cm}}\right)\left(3\frac{\text{mg}}{\text{L}}\right) = 2.38 \times 10^{-10}\ \frac{\text{cm}\cdot\text{mg}}{\text{L}\cdot\text{s}}$$

= 0.3003 g/acre/year

③ CONT **OPTION B**

diffusive flux $= D_{eq}(\Delta C/\Delta z)$

$\Delta z = t_{GM} + t_{clay} = 91.5924$

$= (9.49 \times 10^{-7} cm^2/s)\left(\frac{3.3-0\ mg/L}{91.5924 cm}\right) = 3.42 \times 10^{-8} \frac{cm \cdot mg}{L \cdot s}$

$=$ 43.08 g/acre/yr #

Total flux $= 43.08 + 0.3003 = 43.38$ g/acre/yr

$=$ 0.0434 kg/acre/yr (50 acre)

$= 2.169$ kg/year

∴ I would pick option B because it is the closest to the amount unable to exceed at 2.169 kg TCE/year

CIEG436 HW5 #3 for ABET

Environmental

3

	A	B
geomembrane (mil)	30	60
clay (ft)	3.5	3

concentration ppm	3
lechate depth (cm)	30.48

Option A

material	thickness (cm)	K (cm/s)	D (cm^2/sec)
geomembrane	0.0762	1.00E-13	3.00E-09
clay	106.68	1.00E-07	2.00E-06

keq	1.40E-10	cm/sec
Deq	1.36E-06	cm2/sec
delta h	137.2362	cm
delta L	106.7562	cm
advective flux	5.40E-10	cm *mg/ L*s
diffusive flux	3.81E-08	cm *mg/ L*s

sum	3.86E-08	cm *mg/ L*s
	0.05	kg/acre/day
for 50 acres	2.43	kg/day

Option B

material	thickness (cm)	K (cm/s)	D (cm^2/sec)
geomembrane	0.1524	1.00E-12	3.00E-09
clay	91.44	1.00E-07	2.00E-06

keq	5.97E-10	cm/sec
Deq	9.49E-07	cm2/sec
delta h	122.0724	cm
delta L	91.5924	cm
advective flux	2.39E-09	cm *mg/ L*s
diffusive flux	3.11E-08	cm *mg/ L*s

sum	3.35E-08	cm *mg/ L*s
	0.04	kg/acre/day
for 50 acres	2.11	kg/day

I would choose option B as it is closer to the target value of 2 kg/day

4

Z/h (in)	12
Z/h (cm)	30.48
Ctce (mg/L)	3
Ctce (g/cm3)	0.003
acres	50

Case A	t (cm)	K (cm/sec)	De (cm2/sec)	Case B	t (cm)	K (cm/sec	De (cm2/sec)
liner	0.0762	1.00E-14	3.00E-09	liner	0.1524	1.00E-13	3.00E-09
GM	106.68	1.00E-08	2.00E-06	GM	91.44	1.00E-08	2.00E-06
Keq (cm/s)	1.40E-11			Keq (cm/s)	5.97E-11		
Deq (cm2/s)	1.36E-06			Deq (cm2/s)	9.49E-07		
Jwadv (kg/yr)	0.0034			Jwadv (kg/yr)	0.0150		
Jwdiff (kg/yr)	2.4003			Jwdiff (kg/yr)	1.9581		
Total flux (kg/yr)	2.4037			Total flux (kg/yr)	1.9732		

I would pick Option B since Option A released more than 2 kg/yr, and that is not allowed. However, Option B meets the requirement

Codes:

```
Function Keq(tGM, tC, KGM, KC)
   Keq = (tGM + tC) / ((tGM / KGM) + (tC / KC))

End Function

Function Deq(tGM, tC, DGM, DC)
   Deq = (tGM + tC) / ((tGM / DGM) + (tC / DC))

End Function

Function Jw_adv(Keq, tGM, tC, Z, Cw, acre)
   deltah = Z + tGM + tC
   deltaL = tGM + tC
   Jw = Keq * (deltah / deltaL) * Cw
   Jw_adv = Jw * (4 * 10 ^ 7) * (3.15 * 10 ^ 7) * (1 / 1000) * (1 / 1000) * acre

End Function

Function Jw_diff(Deq, Cw, tGM, tC, acre)
   'assume Ci is zero
   deltaz = tGM + tC
   Jwd = Deq * (Cw / deltaz)
   Jw_diff = Jwd * (4 * 10 ^ 7) * (3.15 * 10 ^ 7) * (1 / 1000) * (1 / 1000) * acre

End Function
```